2.Annuities

Such as mortgage and car payments financial obligations require a regular series of periodic payments. Series of regular payments such as these are called *annuities*.

Geometric Series

To find the present value or future value of an annuity, we will need to use the formula for the sum of a geometric series.

$$1 + r + r^2 + ... + r^n = \frac{1 - r^{n+1}}{1 - r}, r \neq 1$$

if $\mid r \mid < 1$, the infinite geometric series converges:

$$1 + r + r^2 + ... + r^n = \frac{1}{1 - r}$$

Exercise: Write R function code that computes the geometric series.

```
geomsum = function(a, r, n) {
x = 0
for(i in 1:(n+1)) x = x + a * r^(i-1)
return(x)
}
geomsum(1,2,3)
```

```
## [1] 15
```

Annuity Immediate

The present value of an immediate annuity with n payments of 1 and interest rate i is denoted by $a_{\overline{n}|i}$ or $a_{\overline{n}|}$.

In this annuity, first payment made at the end of the first year.

$$\begin{aligned} PV &= a_{\overline{n}|} \\ &= v + v^2 + \cdots + v^n \\ &= v\frac{(1 - v^n)}{1_v} = v\frac{1 - v^n}{d} \\ &= v\frac{(1 - v^n)}{iv} \\ &= \frac{(1 - v^n)}{i} \end{aligned}$$

Exercise: Write the function of R code that calculates the present value of immediate annuity.

```
pv_imm_ann = function(a, i, n) {
x = 0
r=1/(1+i)
for(i in 1:n) x = x + a * r^(i)
return(x)
}
pv_imm_ann(1,0.05,10)
```

```
## [1] 7.721735
```

```
pv_imm_ann(100,0.05,10)
```

```
## [1] 772.1735
```

The future value of the unit annuity immediate with n payments is denoted by $s_{\overline{n}|}$. It is the sum of the future values of the individual payments of 1.

$$s_{\overline{n}|} = (1+i)^{n-1} + (1+i)^{n-2} + \cdots + (1+i) + 1$$
$$s_{\overline{n}|}(1+i)^n a_{\overline{n}|} = \frac{(1+i)^n - 1}{i}$$

Exercise: Write the function of R code that calculates the future value of immediate annuity.

```
fv_imm_ann = function(a, i, n) {
x = 0
r=(1+i)
for(i in 1:n) x = x + a * r^(i-1)
return(x)
}
fv_imm_ann(1,0.05,10)
```

```
## [1] 12.57789
```